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Bianchi Type VI_0 Magnetized Barotropic Bulk Viscous Fluid Massive String Universe in General Relativity

Raj Bali · Anirudh Pradhan · Hassan Amirhashchi

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Abstract A model of a cloud formed by massive strings is studied in the context of the usual general relativity. This model is used as a source of Bianchi type VI_0 massive with magnetic field and bulk viscosity. To get a determinate model, we assume that the expansion (θ) in the model is proportional to the shear (σ) and also the fluid obeys the barotropic equation of state. The behaviour of the models from physical and geometrical aspects in presence and absence of magnetic field and bulk viscosity is discussed.

Keywords Massive string · Bianchi type VI_0 · Viscous models · Magnetic field

1 Introduction

One of the outstanding problems in cosmology today is developing a more precise understanding of structure formation in the universe, that is, the origin of galaxies and other large-scale structures. Existing theories for the structure formation of the Universe fall into two categories, based either upon the amplification of quantum fluctuations in a scalar field during *inflation*, or upon symmetry breaking phase transition in the early Universe which leads to the formation of *topological defects* such as domain walls, cosmic strings, monopoles, textures and other 'hybrid' creatures. Cosmic strings play an important role in the study of the early universe. These arise during the phase transition after the big bang explosion as the

R. Bali
Department of Mathematics, University of Rajasthan, Jaipur 302 004, India
e-mail: balir5@yahoo.co.in

A. Pradhan (✉)
Department of Mathematics, Hindu Post-graduate College, Zamania 232 331, Ghazipur, India
e-mail: pradhan@iucaa.ernet.in

H. Amirhashchi
Department of Physics, Mahshahr Islamic Azad University, Mahshahr, Iran
e-mail: hashchi@yahoo.com

temperature goes down below some critical temperature as predicted by grand unified theories (Zel'dovich et al. [38, 39]; Kibble [12, 13]; Everett [10]; Vilenkin [28]). It is believed that cosmic strings give rise to density perturbations which lead to formation of galaxies (Zel'dovich [36]). These cosmic strings have stress energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings. The general treatment of strings was initiated by Letelier [17, 18] and Stachel [25].

The occurrence of magnetic fields on galactic scale is well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged. Several authors (Zel'dovich [37]; Harrison [11]; Misner et al. [20]; Asseo and Sol [1]; Pudritz and Silk [23]; Kim et al. [14]; Perley and Taylor [22]; Kronberg et al. [15]; Wolfe et al. [33]; Kulsrud et al. [16] and Barrow [7]) have pointed out the importance of magnetic field in different context. As a natural consequences, we should include magnetic fields in the energy-momentum tensor of the early universe. The string cosmological models with a magnetic field are also discussed by Banerjee et al. [8], Chakraborty [9], Tikekar and Patel [26, 27], Patel and Maharaj [21], Singh and Singh [24]. On the other hand, the matter distribution is satisfactorily described by perfect fluid due to the large scale distribution of galaxies in the universe. However, the observed physical phenomena such as the large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation, suggest analysis of dissipative effects in cosmology. Furthermore, there are several processes which are expected to give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during the recombination era. Bulk viscosity is associated with the GUT phase transition and string creation. This motivates us to study cosmological bulk viscous fluid string model in presence of magnetic field. We had two main reasons to study the above-mentioned models. First, as a test of consistency, for some particular field theories based on string models and other models that use strings as basic elements we must have a reasonable behaviour of the gravitational field produced by these strings. Second, we point out that the universe can be represented by collection of extended objects (galaxies). So a "string dust" cosmology gives us a model to investigate this fact.

Recently Bali et al. [2–6], Yadav et al. [34, 35] have investigated Bianchi type I, II, III, V, and IX magnetized string cosmological models in presence of bulk viscosity. Bulk viscous string cosmological models are also studied by Wang [29–32]. Tikekar and Patel [27] have investigated some solutions for Bianchi type VI_0 cosmology in presence and absence of magnetic field. In this paper, we have obtained some Bianchi type VI_0 string cosmological models in presence and absence of magnetic field and bulk viscosity. This paper is organized as follows: The metric and field equations are presented in Sect. 2. In Sect. 3, we deal with the solution of the field equations in presence of viscous fluid and magnetic field. In Sect. 4, we have described some geometric and physical behaviour of the model. Section 5 includes the solution in absence of magnetic field whereas in Sect. 6, we have given the solution in absence of bulk viscosity. In the last Sect. 7, concluding remarks are given.

2 The Metric and Field Equations

We consider the Bianchi Type VI_0 metric in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)e^{2x}dy^2 + C^2(t)e^{-2x}dz^2. \tag{1}$$

The energy-momentum tensor for a cloud of strings in presence of bulk viscosity and magnetic field has the form

$$T_{ik}=(\rho+p)v_iv_k+pg_{ik}-\lambda x_ix_k-\zeta\theta(g_{ik}+v_iv_k)+\left[g^{lm}F_{il}F_{km}-\frac{1}{4}g_{ik}F_{lm}F^{lm}\right], \quad (2)$$

where v_i and x_i satisfy conditions

$$v^iv_i=-x^ix_i=-1, \qquad v^ix_i=0. \quad (3)$$

In (2), p is isotropic pressure, ρ is rest energy density for a cloud strings, λ is the string tension density, F_{ij} is the electromagnetic field tensor, x^i is a unit space-like vector representing the direction of string, and v^i is the four velocity vector satisfying the relation

$$g_{ij}v^iv^j=-1. \quad (4)$$

Here, the co-moving coordinates are taken to be $v^1=0=v^2=v^3$ and $v^4=1$ and $x^i=(\frac{1}{A},0,0,0)$. The Maxwell's equations

$$F_{ij;k}+F_{jk;i}+F_{ki;j}=0, \quad (5)$$

$$F^{ik}_{;k}=0, \quad (6)$$

are satisfied by

$$F_{23}=K(\text{say})=\text{constant}, \quad (7)$$

where a semicolon (;) stands for covariant differentiation.

The Einstein's field equations (with $\frac{8\pi G}{c^4}=1$)

$$R_i^j-\frac{1}{2}Rg_i^j=-T_i^j-\Lambda g_i^j, \quad (8)$$

for the line-element (1) lead to the following system of equations:

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}+\frac{1}{A^2}=-\left[p-\lambda-\zeta\theta-\frac{K^2}{2B^2C^2}\right]-\Lambda, \quad (9)$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}-\frac{1}{A^2}=-\left[p-\zeta\theta+\frac{K^2}{2B^2C^2}\right]-\Lambda, \quad (10)$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}-\frac{1}{A^2}=-\left[p-\zeta\theta+\frac{K^2}{2B^2C^2}\right]-\Lambda, \quad (11)$$

$$\frac{A_4B_4}{AB}+\frac{B_4C_4}{BC}+\frac{C_4A_4}{CA}-\frac{1}{A^2}=\left[\rho+\frac{K^2}{2B^2C^2}\right]-\Lambda, \quad (12)$$

$$\frac{1}{A}\left[\frac{C_4}{C}-\frac{B_4}{B}\right]=0, \quad (13)$$

where the sub indice 4 in A, B, C denotes ordinary differentiation with respect to t. The velocity field v^i is irrotational. The scalar expansion θ and components of shear σ_{ij} are given by

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}, \tag{14}$$

$$\sigma_{11} = \frac{A^2}{3}\left[\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C}\right], \tag{15}$$

$$\sigma_{22} = \frac{B^2}{3}\left[\frac{2B_4}{B} - \frac{A_4}{A} - \frac{C_4}{C}\right], \tag{16}$$

$$\sigma_{33} = \frac{C^2}{3}\left[\frac{2C_4}{C} - \frac{A_4}{A} - \frac{B_4}{B}\right], \tag{17}$$

$$\sigma_{44} = 0. \tag{18}$$

Therefore

$$\sigma^2 = \frac{1}{3}\left[\frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2} - \frac{A_4 B_4}{AB} - \frac{B_4 C_4}{BC} - \frac{C_4 A_4}{CA}\right]. \tag{19}$$

3 Solutions of the Field Equations

The field equations (9)–(13) are a system of five equations with eight unknown parameters A, B, C, ρ, p, λ, ζ and Λ. We need three additional conditions to obtain explicit solutions of the system.

Equation (13) leads to

$$C = mB, \tag{20}$$

where m is an integrating constant.

We first assume that the expansion (θ) is proportional to shear (σ). This condition and (20) lead to

$$\frac{1}{\sqrt{3}}\left(\frac{A_4}{A} - \frac{B_4}{B}\right) = \ell\left(\frac{A_4}{A} + \frac{2B_4}{B}\right) \tag{21}$$

which yields to

$$\frac{A_4}{A} = n\frac{B_4}{B}, \tag{22}$$

where $n = \frac{(2\ell\sqrt{3}+1)}{(1-\ell\sqrt{3})}$ and ℓ are constants. Equation (22), after integration, reduces to

$$A = \beta B^n, \tag{23}$$

where β is a constant of integration. Equations (10) and (12) lead to

$$p = \xi - \frac{K^2}{2B^2C^2} - \left(\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{1}{A^2}\right) - \Lambda, \tag{24}$$

and

$$\rho = \frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{C_4 A_4}{CA} - \frac{1}{A^2} - \frac{K^2}{2B^2C^2} + \Lambda, \tag{25}$$

respectively, where $\zeta\theta = \xi(\text{say}) = \text{constant}$. Now let us consider that the fluid obeys the barotropic equation of state

$$p = \gamma\rho, \tag{26}$$

where γ $(0 < \gamma < 1)$ is a constant. Equations (24) to (26) lead to,

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + (1+\gamma)\frac{A_4 C_4}{AC} + \gamma\left(\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC}\right) - (1+\gamma)\frac{1}{A^2} + (1-\gamma)\frac{K^2}{2B^2C^2} + (1+\gamma)\Lambda - \xi = 0. \tag{27}$$

Equation (27) with the help of (20) and (23) reduces to

$$2B_{44} + \frac{2(n^2 + 2\gamma n + \gamma)}{(n+1)}\frac{B_4^2}{B^2} = \frac{2(1+\gamma)}{\beta^2 B^{2n-1}} + \frac{(1-\gamma)K^2}{m^2 B^3} + 2\ell_0 B, \tag{28}$$

where $\ell_0 = (1+\gamma)\Lambda - \xi$.
Let us consider $B_4 = f(B)$ and $f' = \frac{df}{dB}$. Hence (28) takes the form

$$\frac{d}{df}(f^2) + \frac{2\alpha}{B} f^2 = \frac{2(1+\gamma)}{\beta^2 B^{2n-1}} + \frac{(1-\gamma)K^2}{m^2 B^3} + 2\ell_0 B, \tag{29}$$

where $\alpha = \frac{(n^2+2n\gamma+\gamma)}{(n+1)}$. Equation (29) after integrating reduces to

$$f^2 = \frac{2(1+\gamma)B^{-2n+2}}{\beta^2(2\alpha - 2n + 2)} + \frac{(1-\gamma)K^2}{2m^2(\alpha-1)} + \frac{\ell_0 B^2}{(\alpha+1)} + MB^{-2\alpha}, \quad \gamma \neq 1. \tag{30}$$

To get deterministic solution in terms of cosmic time t, we suppose $M = 0$. In this case (30) takes the form

$$f^2 = aB^{-2(n-1)} + bB^{-2} + NB^2, \tag{31}$$

where

$$a = \frac{2(1+\gamma)}{\beta^2(2\alpha - 2n + 2)}, \qquad b = \frac{(1-\gamma)K^2}{2m^2(\alpha-1)}, \qquad N = \frac{(1+\gamma)\Lambda - \xi}{(\alpha+1)}.$$

Therefore, we have

$$\frac{dB}{\sqrt{aB^{-2(n-1)} + bB^{-2} + NB^2}} = dt. \tag{32}$$

To get deterministic solution, we assume $n = 2$. In this case integrating (32), we obtain

$$B^2 = \sqrt{(a+b)}\frac{\sinh(2\sqrt{N}t)}{\sqrt{N}}. \tag{33}$$

Hence, we have

$$C^2 = m^2\sqrt{(a+b)}\frac{\sinh(2\sqrt{N}t)}{\sqrt{N}}, \tag{34}$$

$$A^2 = \beta^2(a+b)\frac{\sinh^2(2\sqrt{N}t)}{N}, \tag{35}$$

where $N > 0$ without any loss of generality.
Therefore, the metric (1) in presence of magnetic field and bulk viscosity, reduces to the form

$$\begin{aligned} ds^2 &= -dt^2 + \beta^2(a+b)\frac{\sinh^2(2\sqrt{N}t)}{N}\,dx^2 \\ &\quad + \sqrt{(a+b)}\frac{\sinh(2\sqrt{N}t)}{\sqrt{N}}e^{2x}\,dy^2 + m^2\sqrt{(a+b)}\frac{\sinh(2\sqrt{N}t)}{\sqrt{N}}e^{-2x}dz^2. \end{aligned} \tag{36}$$

4 The Geometric and Physical Significance of Model

The pressure (p), energy density (ρ), the string tension density (λ), the particle density (ρ_p), the scalar of expansion (θ), the shear tensor (σ) and the proper volume (V^3) for the model (36) are given by

$$\begin{aligned} p &= \left[\frac{N}{\beta^2(a+b)} - \frac{K^2N}{2m^2(a+b)}\right]\coth^2(2\sqrt{N}t) + \xi \\ &\quad + \left[\frac{K^2}{2m(a+b)} - \frac{1}{\beta^2(a+b)} - 8\right]N - \Lambda, \end{aligned} \tag{37}$$

$$\begin{aligned} \rho &= \left[5N - \frac{N}{(a+b)}\left(\frac{K^2}{2m^2} + \frac{1}{\beta^2}\right)\right]\coth^2(2\sqrt{N}t) \\ &\quad + \frac{N}{(a+b)}\left(\frac{K^2}{2m^2} + \frac{1}{\beta^2}\right) + \Lambda, \end{aligned} \tag{38}$$

where $p = \gamma\rho$ is satisfied by (27).

$$\begin{aligned} \lambda &= \left[\frac{2N}{\beta^2(a+b)} - \frac{K^2N}{m^2(a+b)} - N\right]\coth^2(2\sqrt{N}t) \\ &\quad + \left\{\frac{K^2N}{m^2(a+b)} - \frac{2N}{\beta^2(a+b)} - 4N\right\}, \end{aligned} \tag{39}$$

$$\begin{aligned} \rho_p = \rho - \lambda &= \left[\frac{K^2N}{2m^2(a+b)} - \frac{3N}{\beta^2(a+b)} + N\right]\coth^2(2\sqrt{N}t) \\ &\quad + 9N + \left\{\frac{3N}{\beta^2(a+b)} - \frac{K^2}{2m^2(a+b)}\right\}, \end{aligned} \tag{40}$$

$$\theta = 4\sqrt{N}\coth(2\sqrt{N}t), \tag{41}$$

$$\sigma = \sqrt{\frac{N}{3}}\coth(2\sqrt{N}t), \tag{42}$$

$$V^3 = \frac{\beta m(a+b)}{N}\sinh^2(2\sqrt{N}t). \tag{43}$$

From (30) and (31), we obtain

$$\frac{\sigma}{\theta} = \text{constant}. \tag{44}$$

The deceleration parameter is given by

$$q = -\frac{\ddot{R}/R}{\dot{R}^2/R^2} = -\left[\frac{\frac{8N}{3} - \frac{8N}{9}\coth^2(2\sqrt{N}t)}{\frac{16N}{9}\coth^2(2\sqrt{N}t)}\right]. \tag{45}$$

From (45), we observe that

$$q < 0 \quad \text{if } \coth^2(2\sqrt{N}t) < 3$$

and

$$q > 0 \quad \text{if } \coth^2(2\sqrt{N}t) > 3.$$

From (38), $\rho \geq 0$ implies that

$$\coth^2(2\sqrt{N}t) \leq \left[\frac{\frac{N}{(a+b)}\left(\frac{K^2}{2m^2} + \frac{1}{\beta^2}\right) + \Lambda}{\frac{N}{(a+b)}\left(\frac{K^2}{2m^2} + \frac{1}{\beta^2}\right) - 5N}\right]. \tag{46}$$

Also from (40), $\rho_p \geq 0$ implies that

$$\coth^2(2\sqrt{N}t) \leq \left[\frac{\frac{3N}{\beta^2(a+b)} - \frac{K^2}{2m^2(a+b)} + 9N}{\frac{3N}{\beta^2(a+b)} - \frac{K^2 N}{2m^2(a+b)} - N}\right]. \tag{47}$$

Thus the energy conditions $\rho \geq 0$, $\rho_p \geq 0$ are satisfied under conditions given by (46) and (47).

The model (36) starts with a big bang at $t = 0$. The expansion in the model decreases as time increases. The proper volume of the model increases as time increases. Since $\frac{\sigma}{\theta} = \text{constant}$, hence the model does not approach isotropy. There is a Point Type singularity (MacCallum [19]) in the model at $t = 0$. For the condition $\coth^2(2\sqrt{N}t) < 3$, the solution gives accelerating model of the universe. It can be easily seen that when $\coth^2(2\sqrt{N}t) > 3$, our solution represents decelerating model of the universe.

5 Solutions in Absence of Magnetic Field

In absence of magnetic field, i.e. when $b \to 0$ i.e. $K \to 0$, we obtain

$$B^2 = 2\sqrt{2}\,\frac{\sinh(2\sqrt{N}t)}{2\sqrt{N}},$$

$$C^2 = 2m^2\sqrt{a}\ \frac{\sinh(2\sqrt{N}t)}{2\sqrt{N}},$$

$$A^2 = 4a\beta^2 \frac{\sinh^2(2\sqrt{N}t)}{4N}. \tag{48}$$

Hence, in this case, the geometry of the universe (36) reduces to

$$\begin{aligned} ds^2 &= -dt^2 + 4\beta^2 a \frac{\sinh^2(2\sqrt{N}t)}{4N}\, dx^2 \\ &\quad + 2\sqrt{2}\frac{\sinh(2\sqrt{N}t)}{2\sqrt{N}} e^{2x}\, dy^2 + 2m^2\sqrt{a}\frac{\sinh(2\sqrt{N}t)}{2\sqrt{N}} e^{-2x} dz^2. \end{aligned} \tag{49}$$

The pressure (p), energy density (ρ), the string tension density (λ), the particle density (ρ_p), the scalar of expansion (θ), the shear tensor (σ) and the proper volume (V^3) for the model (49) are given by

$$p = \frac{N}{a\beta^2}\coth^2(2\sqrt{N}t) + \xi - \left(\frac{1}{a\beta^2} + 8\right) N - \Lambda, \tag{50}$$

$$\rho = \left(5N - \frac{N}{a\beta^2}\right)\coth^2(2\sqrt{N}t) + \frac{N}{a\beta^2} + \Lambda, \tag{51}$$

$$\lambda = \left[\frac{2N}{a\beta^2} - N\right]\coth^2(2\sqrt{N}t) - \left\{\frac{2N}{a\beta^2} + 4N\right\}, \tag{52}$$

$$\rho_p = \rho - \lambda = \left[N - \frac{3N}{a\beta^2}\right]\coth^2(2\sqrt{N}t) + 9N + \frac{3N}{\beta^2 a}, \tag{53}$$

$$\theta = 4\sqrt{N}\coth(2\sqrt{N}t), \tag{54}$$

$$\sigma = \sqrt{\frac{N}{3}}\coth(2\sqrt{N}t), \tag{55}$$

$$V^3 = \frac{\beta m a}{N}\sinh^2(2\sqrt{N}t). \tag{56}$$

From (54) and (55), we obtain

$$\frac{\sigma}{\theta} = \text{constant}. \tag{57}$$

From (51), $\rho \geq 0$ implies that

$$\coth^2(2\sqrt{N}t) \leq \left[\frac{\frac{N}{a\beta^2} + \Lambda}{\frac{N}{a\beta^2} - 5N}\right]. \tag{58}$$

Also from (53), $\rho_p \geq 0$ implies that

$$\coth^2(2\sqrt{N}t) \leq \left[\frac{\frac{3N}{a\beta^2} + aN}{\frac{3N}{a\beta^2} - N}\right]. \tag{59}$$

Thus the energy conditions $\rho \geq 0$, $\rho_p \geq 0$ are satisfied under conditions given by (58) and (59).

The model (49) starts with a big bang at $t = 0$ and the expansion in the model decreases as time increases. The spatial volume of the model increases as time increases. Since $\frac{\sigma}{\theta} =$ constant, hence the anisotropy is maintained throughout. There is a Point Type singularity (MacCallum [19]) in the model at $t = 0$.

6 Solution in Absence of Viscosity

In absence of bulk viscosity i.e. when $N \to 0$, then we obtain

$$B^2 = 2\sqrt{a}t,$$
$$C^2 = 2m^2\sqrt{a}t, \tag{60}$$
$$A^2 = 4\beta^2 at^2.$$

Hence, in this case, the geometry of the universe (49) reduces to

$$ds^2 = -dt^2 + 4\beta^2 at^2\, dx^2 + 2\sqrt{at}\, e^{2x}\, dy^2 + 2\sqrt{at}\, e^{-2x}\, dz^2. \tag{61}$$

The pressure (p), energy density (ρ), the string tension density (λ), the particle density (ρ_p), the scalar of expansion (θ), the shear tensor (σ) and the proper volume (V^3) for the model (61) are given by

$$p = \frac{1}{4a\beta^2 t^2} - \Lambda, \tag{62}$$

$$\rho = -\frac{1}{4a\beta^2 t^2} + \Lambda, \tag{63}$$

$$\lambda = \left(\frac{2}{a\beta^2} - 1\right)\frac{1}{t^2}, \tag{64}$$

$$\rho_p = \rho - \lambda = -\frac{3}{4a\beta^2 t^2} + \frac{1}{4t^2} + \Lambda, \tag{65}$$

$$\theta = \frac{2}{t}, \tag{66}$$

$$\sigma = \frac{1}{2\sqrt{3}}\frac{1}{t}, \tag{67}$$

$$V^3 = 4a\beta m t^2. \tag{68}$$

From (66) and (67), we obtain

$$\frac{\sigma}{\theta} = \text{constant}. \tag{69}$$

From (63), $\rho \geq 0$ implies that

$$-\frac{1}{2\beta\sqrt{a\Lambda}} < t < \frac{1}{2\beta\sqrt{a\Lambda}}. \tag{70}$$

Also from (65), $\rho_p \geq 0$ implies that

$$(a\beta^2 - 3) + 4\Lambda t^2 \geq 0. \tag{71}$$

Thus the energy conditions $\rho \geq 0$, $\rho_p \geq 0$ are satisfied under conditions given by (70) and (71).

The model (61) in absence of magnetic field and bulk viscosity, starts with a big bang at $t = 0$ and the expansion in the model decreases as time increases. The spatial volume of the model increases with time. The string tension λ decreases with time. We also observe that $\lambda > 0$ if $\frac{2}{a} > \beta^2$ and $\lambda < 0$ if $\frac{2}{a} < \beta^2$. Since $\frac{\sigma}{\theta} =$ constant, hence the anisotropy is maintained throughout. There is a Point Type singularity (MacCallum [19]) in the model at $t = 0$.

7 Concluding Remarks

Some Bianchi type VI_0 massive string cosmological models with a bulk viscous fluid as the source of matter are obtained in presence and absence of magnetic field. Generally, the models are expanding, sheering and non-rotating. In presence of bulk viscosity it represents an accelerating universe during the span of time mentioned below (45) as decelerating factor $q < 0$ and it represents decelerating universe as $q > 0$. All the three models obtained in the present study have a Point Type singularity at $t = 0$.

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